Ensuring the success of biological control of mosquito-borne diseases by bacteria Wolbachia, through almost global stabilizing feedback

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The use of bacteria Wolbachia is a promising method currently considered to block transmission of diseases such as dengue fever, chicungunha and yellow fever, whose area is currently spreading. While full scale experiments are presently organized in Australia and Brazil, systematic procedures for introduction of mosquitos infected by the bacteria in a healthy population are still to be studied. This is a central question, with heavy impact on the cost and efficiency.

The aim of this talk is the presentation of a method aimed at reducing the number of released mosquitos, and thus the treatment cost, without jeopardizing the success of the introduction of the bacteria — something which could happen e.g. in case of underestimation of the initial population size. We provide a model describing the interactions between healthy and infected populations. The original system is shown to have two stable equilibria, corresponding to Wolbachia-free and complete infestation situations. Taking advantage of the fact that measurements are carried out during the whole release period, the use of control theory techniques is possible. A simple feedback law is proposed, that stipulates the amount of mosquitoes to be introduced at every time instant, and is shown to have the capacity to asymptotically settle the bacteria. Up to our knowledge, this is the first attempt to use feedback for introduction of Wolbachia in a population of arthropods.

The key arguments are based on the theory of monotone systems [2, 3], extended to analyze the asymptotic behavior of input-output monotone systems closed by negative feedback [1, 4, 7]. Due to bistability, the considered input-output system has multivalued static characteristics, but the existing results [5, 6] are unable to prove almost-global stabilization, so a proper analysis has to be set up.

References

[1] J.-L. Gouzé, A criterion of global convergence to equilibrium for differential systems with an application to Lotka-Volterra systems, *Inria research report RR-0894* (1988)

[2] M.W. Hirsch, Stability and convergence in strongly monotone dynamical systems. *J. reine angew. Math* **383** (1988) pp. 1–53

[3] H.L. Smith, *Monotone dynamical systems: an introduction to the theory of competitive and cooperative systems*, Mathematical surveys and monographs 41, American Mathematical Society (1995)

[4] D. Angeli and E.D. Sontag, Monotone control systems, *IEEE Transactions on Automatic Control* **48** (2003) pp. 1684–1698

[5] M. Malisoff and P.D. Leenheer, A small-gain theorem for monotone systems with multi-valued input- state characteristics, *IEEE Transactions on Automatic Control* **41** no. 2 (2006) pp. 287–292

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[6] T. Gedeon and G. Hines, Multi-valued characteristics and Morse decompositions. *Journal of Differential Equations* **247** no. 4 (2009) pp. 1013–1042

[7] G. Enciso, Fixed points and convergence in monotone systems under positive or negative feedback, *International Journal of Control* **87** no. 2 (2014), pp. 301–311